PRECAST CONCRETE CONNECTIONS
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Precast Concrete Connections

Synopsis: With the rapid development of building & construction industry worldwide especially in developed countries, there has been a significant trend to predominantly use prefabrication prefinished & precast construction methods due to economy in construction cost, manpower and time. Due to the dedication and diligence of all related stakeholders in C&S industry and Authorities, Singapore is also now re-known globally as one of the leading, reliable & reputable countries in the civil & structural engineering practices, standards and developments globally.

With that there is an important need for C&S practitioners to keep up to date to the latest developments and advances especially in precast and prefabrication technologies worldwide. This presentation will explore and study on the recent advances in precast connection systems adopted or researched for building structures in developed countries worldwide.

(This part one of series of lectures will focus on EC2 design so it is more related to local & regional content exclude seismic design of precast connections i.e. not including ACI & EC8 design. In low seismic region i.e. low ductility structures Singapore’s NA to EC8 mentions provisions of EC2 and EC3 & EC4 is sufficient)

*Slides are only for educational purposes for this IES Seminar. Detail references should be made to the Building Codes in actual design process and signing QP’s calculation.*
DCL steel reinforcement detailing for reinforced concrete structures would follow the requirements of SS EN 1992-1-1 (Design of concrete structures – General rules and rules for buildings) and in conjunction with Clause 5.3.2(1)P of SS EN 1998-1. For buildings of structural steel or composite construction, the element design shall follow the relevant parts of SS EN 1993 (Design of steel structures) or SS EN 1994 (Design of composite structures) respectively.
Since 2000 till now in 2018, precast concrete elements and structures worldwide have become:

(i) Taller (e.g. 36-storey skeletal precast frame) and 54-storey wall frame (the Netherlands)
(ii) Longer (e.g. 50 m long prestressed concrete beams)
(iii) Deeper (e.g. 1000 mm deep prestressed hollow core floor units produced in Italy in 2014)
(iv) Shallower (e.g. span/depth ratio approaching 40 for prestressed composite and continuous beams)
(v) Stronger (e.g. grade C90/105 used in columns in buildings such as in 36-storey skeletal frame, Belgium)

These advancements have been complimented by an increase in the available literatures & research reports but also from other books from Netherlands, Germany, Brazil, United Kingdom, and several bulletins from fib Commission 6 on Prefabrication, together with 8 European product standards covering a wide range precast concrete elements (hollow core floor slabs, walls, stairs, etc.).
The design and construction of joints and connections is the **most important** consideration in precast concrete structures.

Their purpose is to transmit forces between structural members and/or to provide stability and robustness.

**Within a single connection**, there may be several **load-transmitting joints**, and so it is first necessary to distinguish between a ‘joint’ and a ‘connection’.

A ‘**joint**’ is the action of forces (e.g. tension, shear, compression) that takes place at the interface between two (or more) structural elements.

The definition of a ‘**connection**’ is the action of forces (e.g. tension, shear, compression) and/or moments (bending, torsion) through an assembly comprising one (or more) interfaces.

The design of the connection is therefore a function of both the structural elements and of the joints between them.

**Recent advances in precast connection systems adopted or researched for building structures in developed countries worldwide does not differ from basic fundamentals**
Definition of 'joint' and 'connection'.
The most commonly used methods of connection analysis in precast are:

1. Strut-and-tie, for the transfer of bearing forces
2. Coupled joint, for the transfer of bearing forces and/or bending and/or torsional moments
3. Shear friction or shear wedging, for the transfer of shear with or without compression
One of most commonly used methods of connection analysis

Strut-and-tie, for the transfer of bearing forces
Two of most commonly used methods of connection analysis:

- **Column-Column Splice**: A coupled joint, for the transfer of bearing forces and/or bending and/or torsional moments.

- **Shear transmission through shear keys**: Shear friction or shear wedging, for the transfer of shear with or without compression.
Compression Joint. Compressive force is transmitted between precast concrete components either by direct bearing, or through an intermediate medium such as in situ mortar or concrete. The distinction is made depending on tolerances and the importance of the accuracy of the load transfer location.
Compression Joint.

Force and stress limitation in compressive joints where (a) bearing width is equal to the supporting element and (b) bearing width is narrower than the supporting element.
Compression Joint.

Types of bearings.
Shear forces can be transferred between concrete elements by one, or more, of the following methods:

1. **Shear adhesion and bonding** (when cast in situ concrete is placed against a precast concrete surface, adhesive bond develops in the fresh cement paste in the tiny crevices and pores in the mature concrete)
2. **Shear friction**
3. **Shear keys**
4. **Dowel action**
5. **Mechanical devices**
Shear Joint.

Shear forces can be transferred between concrete elements by one, or more, of the following methods:

2. Shear friction
**Shear Joint.**

Shear forces can be transferred between concrete elements by one, or more, of the following methods:

3. **Shear Key**
Shear Joint.

Shear forces can be transferred between concrete elements by one, or more, of the following methods:

4. Dowel Action

Dowel action for shear resistance (a) definitions and (b) principles.
Shear Joint.

Shear forces can be transferred between concrete elements by one, or more, of the following methods:

4. Mechanical devices

The most common form of mechanical connection is the welded plate or bar shown in Figure.

This may be cast into elements such as at the edges of double-tee floor units.

The ultimate shear capacity of the welded plate joint is the least of (a) The pull-out resistance of the embedded plate (b) The weld capacity of the holding bars to the embedded plate, or (c) The shear capacity of the intermediate plate or bar.
Tension Joint.

Plan

Elevation

Tension joint using: (a) lapped loops and (b) bond resistance.
Tension Joint.
**Tension Joint.**

**By Bolting**

**By Welding**

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**Minimum weld lengths to develop full strength in lapped bars**

<table>
<thead>
<tr>
<th>Bar diameter (mm)</th>
<th>Weld depth (mm)</th>
<th>Weld length (mm)</th>
<th>Nominal length (mm)</th>
</tr>
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<tbody>
<tr>
<td>12</td>
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\[ f_{ik} = 250 \text{ N/mm}^2; \gamma_m = 1.15; \rho_{yeb} = 220 \text{ N/mm}^2 \text{ for grade E275 electrode } \gamma_M = 1.15. \]
In early 1950s in UK, adoption of H-frame was done, whereby pin-jointed site connections were made near the positions of frame contraflexure; i.e. mid-column height and 0.2 × beam span. The solution removed one problem by shifting the connections to points of zero (or small) bending moment, but caused other difficulties elsewhere in manufacturing and transportation by creating large, two-dimensional units.
The main cost elements are fabrication and material costs, and those rise steeply with increases in connector capacity. Precast designers prefer the complicated aspects of the connector to be under the control of the factory, reducing site operations to simple dowelling, bolting or welding.

Stanton et al. (1986) produced a special study for the PCI on moment-resistant and simple connections, and stated:

The economic and functional success of a precast concrete structure depends to a great extent on the configuration and properties of its inter-element connections. Flexibility of the connections affects the distribution of creep, thermal and shrinkage strains and determines joint performance over time. The strength of the connections remaining after the demands of volume change, gravity loads and dimensional or alignment corrections have been satisfied determines whether the structure will deform permanently under action of extreme wind or seismic forces. Ductility of the connections determines whether the permanent deformations will take place by a safe redistribution of load and dissipation of energy or by brittle connection failure.

This statement captures the essence of precast frame design in knowing which effects are to be considered in design, and not simply carrying out the mechanics of stress equilibrium and strain compatibility.
For instance, figure below shows a better solution: a ‘scarf joint’, approximately 0.15L – say, 600 to 750 mm from the column – reduces the bending moment in the beam by around 40 per cent, resulting in a shallower beam. The scarf joint may be positioned at one end of the beam, or at both ends. This method of design has been used in multi-storey car parks in order to save headroom. The most robust solution was found in a mechanical connection, brought about by relying on the precast concrete to firmly anchor rolled or fabricated steel sections in position so that a direct steel-to-steel joint was made.
4 Types Of Precast Connections:

(1) beam-to-slab connections
(2) beam-to-column connections
(3) wall-to-frame connections
(4) column splices, including to foundations.

The four rules for satisfactory joint design are that:

(1) The components can resist ultimate design loads in a ductile manner.
(2) The precast members can be manufactured economically and be erected safely and speedily.
(3) The manufacturing and site erection tolerances do not adversely affect intended structural behaviour, or are catered for in a ‘worst case’ situation.
(4) The final appearance of the joint must satisfy visual, fire and environmental requirements.
Types of beam-to-column connections.

Discontinuous beams need not all be at the same level.
Continuous columns

Discontinuous beams must be at same level
Continuous beams at connection
Discontinuous single storey columns

Type I

e.g. Haunch

e.g. Corbel

Type II
Typical examples of Type I beam-column connections (PCI, 2004)
Typical examples of Type II beam-column connections (PCI, 2004)
The Dywidag Ductile Connector (DDC) system was proposed by Rockwin Corporation in 1995. The connection was usually applied at the junction between the precast column and the precast beam. It was reported that the DDC system was capable of providing sufficiently high flexural capacity, and was also advantageous in fast erection and excellent seismic resistance.
HDB’s precast column is usually fabricated in a rectangular shape resulting in larger flexural rigidity in the frame direction. The cross-section length of column, \( h \), usually varies from 200mm to 2000mm and the cross-section width, \( b \), varies from 200mm to 400mm in practice. For the precast beam, a composite beam is usually formed after the cast in place slab strip. The overall height of beam varies from 300mm to 1000mm and the width of beam varies from 200mm to 1000mm in practice. Due to constraints imposed on dimensions by several factors (clear head room requirement, low typical storey height, erection crane capacity, etc.), precast components of significantly larger dimensions are seldom used. The typical column dimensions \((h \times b)\) adopted in HDB practice is 1000mm\(\times\)300mm and the precast beam of dimensions 300mm in width and 500mm in depth are typically adopted. In addition, precast columns with a visible corbel are not acceptable due to clear head room requirement for residential buildings. The clear span of the beam in HDB apartments commonly ranged from 4m to 6m.
1. The proposed DfD M-R beam–column connection is shown to be capable of providing adequate moment resistance for use in multi-storey RC apartment blocks in Singapore. The proposed connection also showed good ductility when assessed based on the CDF values obtained.

2. Certain design adjustments was made to ensure that the serviceability limit state of cracking can be satisfied through the provision of a topping with continuity rebars (3H12). In general, the moment capacities corresponding to when a maximum crack width of 0.3 mm was observed during testing reached about 50–65% of the ultimate moment capacities.

3. Increasing the size of the specimen especially the depth is effective in increasing the ultimate moment capacity of the proposed connection. Under quasi static loading all the specimens tested performed similarly in terms of crack propagation, failure mode and the ductile behaviour.

Note: Other commercially marketed systems have also been proposed in the Netherlands, e.g. BESTCON 30, CD20 and MATRIXBOUW.
Importance of Basics of Precast Connections Design Concepts acting as fundamentals of development of *Advances in Precast Connections.*
PIN-JOINTED CONNECTIONS

Pinned connections are used extensively in precast structures as they may be formed in the simplest manner by element to element bearing. The very nature of precast construction lends itself to forming simply supported connections in order to avoid flexural continuity across the ends of individual elements. For this reason, they are often referred to as ‘joints’ as they tend to involve one bearing surface only.
PIN-JOINTED CONNECTIONS

Simply supported slabs on beams or walls
MOMENT-RESISTING (MR) CONNECTIONS

Principles of negative moment-resisting connections.

Positions of moment-resisting connections in precast skeletal structures.
MOMENT-RESISTING CONNECTIONS

MOMENT-RESISTING CONNECTIONS

Grouted joints for moment resistance

- Local check out for grout or small agg. concrete
- Levelling shims
- Small lengths of bar flat or angle to make weld between main bars

Column moment continuity by welding small angles or plates to projecting rebars in lower and upper columns.
Eg. 7.115  Welded plate splice design

Calculate axial load capacity of the welded plate splice shown in Figure 7.115, assuming that the bending moment is zero. Assume the edge distance to the holes in the plate = 20 mm.

Use grade C30/37 for in situ concrete, $f_{ck} \{ f_y \} = 250 \text{ N/mm}^2$ in the upper bars, $f_{ck} \{ f_y \} = 500 \text{ N/mm}^2$ in the lower threaded bars, 35 mm cover to the plate, grade S275 [43] steel plate and weld.

Solution

Step 1: During construction
Assume 25 mm threaded bar, then $X = 230/2 - 20 - 25/2 = 82.5 \text{ mm}$

$b = \sqrt{2} \times 82.5 = 116.7 \text{ mm}$

Plate bending capacity $M_R = 275 \times 116.7 \times 20^2 \times 10^{-3}/4 = 3209 \text{ kNmm}$

Figure 7.115  Details for Exe
Example 7.115

Welded Plate Splice Design

MOMENT-RESISTING CONNECTIONS

Eurocode

BS8110

\[ M_p = F_i X / \sqrt{2} \]
Hence \( F_i = 3209/58.35 = 55 \text{ kN} \)
Plate shear capacity \( V_p = 165 \times 116.7 \times 20 \times 10^{-3} = 385 \text{ kN} \)
Hence \( F_i > V_p > 55 \text{ kN} \) bending capacity
Ultimate axial capacity of splice = \( 4 \times 55 = 220 \text{ kN} \).

Threaded bar \( A_t = \frac{55 \times 10^4}{0.87 \times 500} = 126.4 \text{ mm}^2 \)
Use H16 bars (157 mm\(^2\)) threaded to M16.
Welded bar \( A_w = \frac{55 \times 10^9}{0.87 \times 250} = 252.8 \text{ mm}^2 \)
Use R20 welded bars (314 mm\(^2\)).

Weld capacity
Use penetration \( x = 0.5 \times 20 = 10 \text{ mm} \)
Contact area = \( \pi \times 20 \times 10 = 628 \text{ mm}^2 \)
Weld strength = \( 628 \times 220 \times 10^{-3} = 138.1 \text{ kN} \) \( \{ 628 \times 215 \times 10^{-3} = 135.1 \text{ kN} \} > 55 \text{ kN} \) required.

Step 2: After concreting
Splice designed as an ordinary rc column
For the welded bars, \( d/h = 0.857 \)
Use \( d/h = 0.15, f'_c = 30 \) and \( f_k = 250 \text{ N/mm}^2 \)
\( \text{For maximum eccentricity for 3 m high column} \)
\( c_e = 20 \text{ mm, and } A_t = 1257 \text{ mm}^2 \)
\( N_{bd}/f_k b h = 0.57 \)
\( N_{bd} = 1540 \text{ kN} \)
For the threaded bars, \( d/h = 0.85 \)
Use \( d/h = 0.15 \) and \( f_k = 500 \text{ N/mm}^2 \) and \( A_t = 628 \text{ mm}^2 \)
\( N_{bd}/f_k b h = 0.57 \)
\( N_{bd} = 1540 \text{ kN} \)
Limiting ultimate axial capacity = \( 1540 \text{ kN} \)

BS 8110, Part 1, eq. 38 gives
\( N = 0.4 f_{c3} A_t + 0.75 f_k A_c \)
For the welded bars
\( N = (0.4 \times 37 \times 88743 + 0.75 \times 250 \times 1257) \times 10^{-3} = 1548 \text{ kN} \)
For the threaded bars
\( N = (0.4 \times 37 \times 89372 + 0.75 \times 500 \times 628) \times 10^{-3} = 1437 \text{ kN} \)
Capacity of concrete in contact with plate, using 0.8 \( f_{cu} \) and ignoring cover concrete
\( N = (0.8 \times 37 \times 230^2) \times 10^{-3} = 1565 \text{ kN} \)
Limiting ultimate axial capacity = \( 1647 \text{ kN} \)
Example 7.116
Grouted Sleeve Splice Design

Cover to sleeve = 40

T25 bar in 50 dia. sleeve

Grout grade C30

Figure 7.116
Example 7.116

Grouted sleeve splice design

Calculate the ultimate moment of resistance of the column splice shown in Figure 7.116, if the maximum and minimum axial force is 1275 kN and 900 kN respectively. Determine the anchorage length of the projecting bars.

Use grade C30/37 for in situ concrete, $f_{yk} \{f_y\} = 500 \text{ N/mm}^2$.

Solution

Assume main bars in column are 25 mm dia. and links are 10 mm dia.

For minimum effective depth with splice bar touching inside face of sleeve

$$d = 300 - 40 \text{ cover} - (50 - 12.5) = 222.5 \text{ mm}$$

$$\frac{d}{h} = \frac{222.5}{300} = 0.742$$

Use $d_y/h = 0.26$ and $A_s = 1963 \text{ mm}^2$

$$A_s f_{yk}/f_{ck} \cdot bh = 0.364$$
Eg. 7.116
Grouted Sleeve Splice Design

\[
\text{For } \frac{N_{bl}}{f_{ck}bh} = \frac{1275 \times 10^3}{30 \times 300 \times 300} = 0.472 \\
M_{bl} = 0.472 \times 30 \times 300 \times 300^2 = 88.3 \text{kNm} \\
\text{For } \frac{N}{f_{ck}bh} = \frac{900 \times 10^3}{30 \times 300 \times 300} = 0.333 \\
M_{bl} = 0.333 \times 30 \times 300 \times 300^2 = 98.8 \text{kNm} \\
\]

Maximum safe moment capacity is \(-88.3 \text{kNm}\)

To determine bond lengths, check stresses in bars.
Axial stress = 14.16 N/mm² 
Bending stress = \(\frac{6M}{bh^2}\) or 10.00 N/mm²

Therefore not all bars are in compression. Supply all bars with tension bond length.

(See cl. 8.4, Anchorage bond length for bars in tension. \(h > 250\) mm.
Casting conditions for top bars = poor
\(f_{cd} = 2.25 \times 0.7 \times 1 \times 0.14 \times 30^{1/3} \)
\(= 2.13 \text{N/mm}^2\)
(See cl. 8.4.4.) \(l_{bd,req} = 0.25 \times 435 \times 25/2.13 = 1276 \text{mm}\)
(See cl. 8.4.4., Table 8.2) \(\alpha_1 = 1\)
\(\alpha_2 = 1 - 0.15 \times (65 - 25)/25 = 0.76\)
\(l_{bd} = 0.76 \times 1276 = 970 \text{mm}\)
Floor connections at load-bearing walls (MR*)

*Note MR denotes Moment Resisting Connections*
Referring to Figure 9.57, the bearing length \( l_b \) should be 75 mm minimum so that the ultimate clamping force \( N_{Ed} \) (acting in the vicinity of the precast slab and not the \textit{in situ} infill) may generate a frictional force \( F = \mu N_{Ed} \) over a sufficient contact length. The wall thickness should therefore be at least 200 mm, allowing a 50 mm wide gap for \textit{in situ} concrete infill. It is assumed that the lever arm from the bearing ledge to the centroid of the tie steel bars is 0.8\( d \), and to the centre of bearing pressure is 0.67\( l_b \). The tensile capacity of the concrete is ignored. The moment capacity of such connections is given by

\[
M_{Rd} = \mu N_{Ed} h + 0.67l_b N_{Ed} + 0.87 f_{yk} A_s 0.8d
\]  

(9.54)

where

- \( \mu \) is the coefficient of friction, taken as 0.7 for concrete–concrete or dry elastomeric surfaces
- \( f_{yk} \) is the yield stress in tie bars of area \( A_s \)
- \( d \) is the effective depth to tie bars from bearing ledge, and \( d > 0.5h \)
Floor connections at load-bearing walls (MR)

Work Example 9.57: Calculate the ultimate moment of resistance in the floor slab to wall connection shown below. The ultimate axial force from the upper wall is 500 kN/m run. Check the compression limit of the infill concrete in the 50 mm gap between the ends of the floor units. Check the limiting moment of resistance of the floor slab itself. Use $f_{ck} = 40 \text{ N/mm}^2$, $f_{cki} = 25 \text{ N/mm}^2$, $f_{yk} = 500 \text{ N/mm}$
Floor connections at load-bearing walls (MR)

Work Example 9.57: Calculate the ultimate moment of resistance in the floor slab to wall connection shown below. The ultimate axial force from the upper wall is 500 kN/m run. Check the compression limit of the infill concrete in the 50 mm gap between the ends of the floor units. Check the limiting moment of resistance of the floor slab itself. Use $f_{ck} = 40$ N/mm$^2$, $f_{cki} = 25$ N/mm$^2$, $f_{yk} = 500$ N/mm$^2$ and $\mu = 0.7$.

Solution
Consider a 1 m wide slab.

Compressive stress beneath the upper wall = $\frac{500 \times 10^3}{1000 \times 200} = 2.5$ N/mm$^2$

Clamping force $N_{Ed} = 2.5 \times 75 \times 1000 \times 10^{-3} = 187.5$ kN

Plane A
At the edge of the bearing $A_x = 377$ mm$^2$/m
$d = 200$ – cover 30 – bar radius 6 = 164 mm

Equation 9.54 $M_{Rd} = [(0.7 \times 187.5 \times 10^3 \times 200) + (187.5 \times 10^3 \times 0.67 \times 75) + (0.87 \times 500 \times 377 \times 0.8 \times 164)] \times 10^{-6} = 57.2$ kNm
Floor connections at load-bearing walls (MR)

Work Example 9.57: Calculate the ultimate moment of resistance in the floor slab to wall connection shown below. The ultimate axial force from the upper wall is 500 kN/m run. Check the compression limit of the infill concrete in the 50 mm gap between the ends of the floor units. Check the limiting moment of resistance of the floor slab itself. Use $f_{ck} = 40 \text{ N/mm}^2$, $f_{cki} = 25 \text{ N/mm}^2$, $f_{yk} = 500 \text{ N/mm}^2$ and $\mu = 0.7$.

Flexural horizontal compressive force in infill $= (57.2 \times 10^6/0.8 \times 164) \times 10^{-3} = 435.9 \text{ kN}$

Corresponding stress assuming rectangular stress block depth $0.4d = 435.9 \times 10^3/1000 \times 0.4 \times 164 = 6.64 \text{ N/mm}^2 < 0.567f_{ck} = 14.17 \text{ N/mm}^2$.

*Plane B*

In the precast floor unit $A_x = 670 \text{ mm}^2/m$

$F_x = 0.87 \times 500 \times 670 \times 10^{-3} = 291.4 \text{ kN}$

$X = 291.4 \times 10^3/0.567 \times 40 \times 1000 \times 0.8 = 16 \text{ mm}$

$z = 152 - 0.4 \times 16 = 145.6 \text{ mm}$

$M_R = 291.4 \times 10^3 \times 145.6 \times 10^{-6} = 42.4 \text{ kNm} < 57.2 \text{ kNm}$ at the edge of bearing.

:. Connection critical in slab as desired.

Hogging moment of resistance $= 42.4 \text{ kN/m width}$.
Beam-to-column face connections (MR)

Welded plate connector

Thin plate is anchored to the beam using large-diameter rebars, typically 25 mm high tensile. The plate is site welded to a projecting steel billet. Expansive infill concrete is used to fill the gap (See left side of column in Figure 9.60a). Providing that the bars are fully anchored to the column or are continuous through the column, the tie steel bars are fully stressed at the ultimate limit state. The beam plate is fully anchored such that the weld at the billet is also fully effective. The compressive strength of the concrete at the bottom of the beam is limited by the strength $f_{ck}$ of the infill concrete. The contribution of the solid steel billet is ignored.

Figure 9.60(a) Moment-resisting beam-to-column connections for (a) negative moment
Beam-to-column face connections (MR)

Steel billet connector
A threaded rod or dowel is site fixed through a hole in the beam and supporting steel billet and secured to a steel angle (or similar) at the top of the beam (right side of column in Figure 9.60a). The annulus around the billet is site grouted. If the tie steel is fully anchored as described earlier, the tie steel bars are fully stressed at the ultimate limit state. The shear strength of the vertical dowel is ignored due to the negligible strength of the bolted angle. The moment due to shear force in the (same) vertical dowel of area is fairly small owing to the nearness of the dowel to the compression zone. The compressive strength of the concrete at the bottom of the beam is limited by the strength $f_{cki}$ of the narrow grouted joint. The contribution of the steel billet is ignored.

Figure 9.60(a) Moment-resisting beam-to-column connections for (a) negative moment
Beam-to-column face connections (MR)

Figure 9.60 Moment-resisting beam-to-column connections for (a) negative moment

Figure 9.60 Moment-resisting beam-to-column connections for (b) positive moment
Beam-to-column face connections (MR)

Work Example 9.62: Calculate the hogging moment of resistance of the beam–column connections shown in Figure 9.62a and 9.62b for the welded plate and billet connectors. In both cases, continuity reinforcement is positioned in the gap between the ends of floor slabs and above the centre line of the beam. Transverse tie steel would be present but is not shown here. Use $f_{cki} = 40 \text{ N/mm}^2$, $f_{yk} = 500 \text{ N/mm}^2$, shear $p_{bq}$ for grade 8:8 bolts = 307 N/mm$^2$, $f_{ywd} = 220 \text{ N/mm}^2$ and cover to top steel = 50 mm.

Figure 9.62 (a) welded plate connector

9.62 (b) billet connector.
Beam-to-column face connections (MR)

Work Example 9.62: Calculate the hogging moment of resistance of the beam–column connection shown in Figure 9.62a and 9.62b for the welded plate and billet connectors. In both cases, continuity reinforcement is positioned in the gap between the ends of floor slabs and above the centre line of the beam. Transverse tie steel would be present but is not shown here. Use $f_{cki} = 40\, \text{N/mm}^2$, $f_{yk} = 500\, \text{N/mm}^2$, shear $p_{bq}$ for grade 8:8 bolts $= 307\, \text{N/mm}^2$, $f_{ywd} = 220\, \text{N/mm}^2$ and cover to top steel $= 50\, \text{mm}$.

In both cases, $d = 500 - 50 - 25/2 = 437\, \text{mm}$

(a) *Welded plate connector*

Weld length actually 80 mm.

$$F_z = [(0.87 \times 500 \times 982) + (220 \times 80 \times 20 \times 0.7)] \times 10^{-3}$$

$$= 427.2 + 248.9 = 676.1\, \text{kN}$$

$$X = \frac{676.1 \times 10^3}{0.567 \times 40 \times 300 \times 0.8} = 124.3\, \text{mm} < 0.6d$$

$z_1$ to the bars $= 437 - 0.4 \times 124.3 = 387\, \text{mm}$

$z_2$ to the weld $= 200 - 0.4 \times 124.3 = 150\, \text{mm}$

$$M_{Re} = 427.2 \times 0.387 + 248.9 \times 0.150 = 202.7\, \text{kNm}$$
Beam-to-column face connections (MR)

Work Example 9.62: Calculate the hogging moment of resistance of the beam–column connection shown in Figure 9.62a and 9.62b for the welded plate and billet connectors. In both cases, continuity reinforcement is positioned in the gap between the ends of floor slabs and above the centre line of the beam. Transverse tie steel would be present but is not shown here. Use $f_{ck} = 40 \text{ N/mm}^2$, $f_{yk} = 500 \text{ N/mm}^2$, shear $p_{bq}$ for grade 8:8 bolts = 307 N/mm$^2$, $f_{ywd} = 220 \text{ N/mm}^2$ and cover to top steel = 50 mm.

See Fig. 9.62(b)

\[
F_c = 0.567f_{ck}b0.8X
\]

\[
F_z = 0.87f_{yk}A_s + p_{bq}A_{sd}
\]

and $F_c = F_z$

hence $X$ and $z$ are determined as before. Finally

\[
M_R = 0.87f_{yk}A_s z_1 + p_{bq}A_{sd}z_2
\]

Assumption: We did not consider Semi-rigid beam-to-column face connections. Current calculations are more conservative. 

Note: Over the past 25 years (since 1990s) around 100 full- or small-scale tests have shown that many typical precast beam–column connections act as semi-rigid joints in flexure, reducing sagging moments in the beam due to imposed gravity load, as well as enhancing the frame action by reducing the buckling height of columns.
Beam-to-Column Connections

Types of beam-to-column connections.

- **Type I**:
  - Continuous columns
  - Discontinuous beams need not all be at the same level
  - e.g. Haunch
  - e.g. Corbel

- **Type II**:
  - Continuous beams at connection
  - Discontinuous single storey columns

Discontinuous beams must be at the same level.
Beam-to-Column Connections

- Top fixing cleat or similar
- Precast beam
- Levelling shims
- Bolt or threaded dowel
- Grout or concrete
- Recess in beam
- Solid or hollow steel section (billet) cast into column

Steel billet beam-to-column (hidden) connector.

Welded plate beam-to-column (hidden) connector. (a) Definitions; (b) actual connector in a 300 mm deep beam.
Beam-to-Column Connections

- Column recess
- Bolted connection between beam and cleat
- Levelling shims
- Steel section or fully anchored sockets cast into column
- Gusseted angle or tee cleat bolted to column
- Cleated beam-to-column (hidden) connector.
Beam-to-Column Connections

Steel box anchored into precast column (single sided version shown)

Sliding plate, typically 20–30 mm thick

Typical depth 400–600 mm

Lip

Notch fits over lip

Steel lined rectangular opening in beam

Narrow plate beam-to-column (hidden) connector: (a) In detail; (b) isometric view.
Beam-to-Column Connections

- Top fixing cleat
- Precast beam
- Projecting rebar in grouted tube
- Levelling and bearing material
- Reinforced concrete shallow corbel cast monolithic with column
- Infill grout
- 45° typically

Shallow corbel beam-to-column (visible) connection.

- Details same as shallow corbel
- Small chamfer
- Reinforced concrete deep corbel cast monolithic with column
- 70° typically

Deep corbel beam-to-column (visible) connection.
Beam-to-Column Connections

Elevation showing 2 beams resting on column

Plan showing 2 beam ends and its interface
Beam-to-Column Connections

Continuous beam to discontinuous column connection.
Beam-to-Column Connections

Conventional Method of PC column and Beam Connection

- Connection of Beam and Column: Cast-in-place concrete

Left Right Vertical (LRV) Installation Precast Method

Beam-Connection Integrated Precast member (LR Beam)

Column PC Member (V-Column)

Column reinforcing bars protrude from bottom

System Developed by Obayashi, Japan
Beam-to-Column Connections

**LRV Installation Method Statement**

1. LR Beam Installation

   * LR Beam able to slide on top of column Level

   * Beam bar at beam end shall be inserted into sleeves in another beam member end

   Completed

   Full PC System / No In-Situ concrete

**V-Column Installation**

* Column reinforcement is pass through penetration holes in the beams. Mortar is used to fill in the joints between pre-cast members - [the main reinforcement penetration holes at LR beam - V column joints] so that respective members are integrated into the framing structure.

Credit Acknowledgement: Obayashi, Japan
Beam-to-Column Connections

Discontinuous beams need not all be at same level. 

Elevation

Elevation

Plan

Prefabricated shear box definitions and principles.

Principles of column insert (billet type connector) design.
Beam-to-Column Connections

Deep recess reinforcement cage using shear stirrups and inclined bars.
Beam-to-Column Connections

Column Insert

There are many types of inserts including:

- Universal column or beam
- Rolled channel, angle or bent plate
- Rolled rectangular hollow section (RHS) or square hollow section (SHS)
- Narrow plate
- Threaded dowels or bolts in steel or plastic tubes
- Bolts in cast-in steel sockets
Beam-to-Column Connections

Column Insert

- Top cover to insert < 150 mm
- Bars area $A_s$ welded to remote end of insert
- Pressure zones overlapping
- Bars welded to sides of insert at front and rear

Additional holding down rebars welded to rear of insert at the top of columns.

Additional rebars welded to insert.
Beam-to-Column Connections

Column Insert

May require additional top bearing plate

Cover to links

Bars welded to sides of insert

Additional rebars welded to sides of narrow plate insert.
Beam-to-Column Connections: Column Insert

Work Example 5

shows the detail of a rolled hollow section steel billet used to support a precast concrete beam. The maximum depth of the RHS is 150 mm. Given that the ultimate beam reaction is 200 kN, calculate the size of the required billet, the confinement reinforcement and the vertical threaded dowel bar. Use $f_{ck} = 40 \text{ N/mm}^2$, $f_{yk} = 500 \text{ N/mm}^2$, $f_y = 275 \text{ N/mm}^2$, $f_{yd}$(grade 8.8 dowel) = 512 N/mm$^2$, $p_{yd}$(grade 4.6 bolts) = 192 N/mm$^2$. Cover to all steel = 30 mm.
Work Example 5

The detail of a rolled hollow section steel billet used to support a precast concrete beam. The maximum depth of the RHS is 150 mm. Given that the ultimate beam reaction is 200 kN, calculate the size of the required billet, the confinement reinforcement and the vertical threaded dowel bar. Use $f_{ck} = 40 \text{ N/mm}^2$, $f_{yk} = 500 \text{ N/mm}^2$, $f_y = 275 \text{ N/mm}^2$, $f_{ybd}$ (grade 8.8 dowel) = 512 N/mm$^2$, $p_{ybd}$ (grade 4.6 bolts) = 192 N/mm$^2$. Cover to all steel = 30 mm.

Solution

Depth of RHS = recess depth 210 mm - 30 mm cover + 20 mm bolt head, washer and tolerance = 160 mm

Try RHS 150 mm deep $\times b_p = 100 \text{ mm wide}$

Initially, try $S_q = 2.0$

Line pressure $p = 0.567 \times 40 \times 20 \times 100 = 4533 \text{ N/mm}$

$L_2 = \frac{200 \times 10^3}{4533} = 44.1 \text{ mm}$

$S_q$ is a biaxial (or often triaxial) confinement
Beam-to-Column Connections: Column Insert

Work Example 5

\[ L_b = 150 - 25 - 30 = 95 \text{ mm}; \quad a = 25 + 95/2 = 72.5 \text{ mm} \]

\[ M_{zz} = 200 \times 10^3 \times (72.5 + 30 + 44.1/2) = 24.91 \times 10^6 \text{ Nmm} \]

The moment inside the column at \( zz \) is

\[ M_{zz} = 0.567 f_{ck} b_p S_q L_3 (L_4 - L_2 - 0.5L_3 - 0.5L_3) \]

\( L_4 \leq 300 - 2 \times \text{cover} 30 = 240 \text{ mm} \)

\( L_4 = b - 2 \times \text{cover} \)

\( L_4 \) is exclusive of the cover concrete to the links;

\[ M_{zz} = 4533L_3(240 - 44.1 - L_3) = 24.91 \times 10^6 \]
Beam-to-Column Connections: Column Insert

Work Example 5

\[ M_{\text{zz}} = 4533L_3(240 - 44.1 - L_3) = 24.91 \times 10^6 \]

Solving \( L_3 = 33.9 \) mm

\[ S_q = \sqrt{\frac{160 \times 138}{100 \times 78}} = 1.68 < 2.0, \]

four further iterations give \( S_q = 1.56 \) (a further four iterations change the final result by 0.4%)

Then

\[ p = 0.567 \times 40 \times 1.56 \times 100 = 3538 \text{ N/mm} \]

\[ L_2 = \frac{200 \times 10^3}{3538} = 56.5 \text{ mm}; \quad M_{\text{zz}} = 26.15 \times 10^6 \text{ Nmm} \text{ and} \quad L_3 = 59.8 \text{ mm} \]
Beam-to-Column Connections: Column Insert

Work Example 5

Check $L_2 + 2L_3 = 176.1 \leq 0.9 \times 240 = 216$ mm

$L_2 + 2L_3 \leq 0.9L_4$

The maximum compressive force occurs below the insert

$$F = 0.567f_{ck}S_q b_p (L_3 + L_2)$$

Total vertical force beneath insert = $3538 \times (56.5 + 59.8) \times 10^{-3} = 411.4$ kN

$$\zeta = \frac{F_{bst}}{V_{Ed}} = 0.25(b - b_p)/b$$

$$\zeta = 0.25(300 - 100)/300 = 0.167 \quad \text{for } b_p/b = 100/300 = 0.3$$

The horizontal bursting force is calculated from the end-block theory

$$F_{bst} = \zeta F$$

$$F_{bst} = 0.167 \times 411.4 = 68.7$$ kN

(to avoid overlapping stress)

(Lateral bursting force coefficient)
Beam-to-Column Connections: Column Insert

Work Example 5

(Area of confinement steel)

\[ A_{bst} = \frac{F_{bst}}{0.87 f_{ywk}} \]

\[ A_{bst} = 68.7 \times 10^3 / 0.87 \times 500 = 158 \text{ mm}^2 \]

\[ L_2 + L_3 = 116.3 \text{ mm} < \text{column } h/2, \text{ then } A_{bst} \text{ refers to one leg only} \]

Use 2 no. H10 (157) at 75 mm spacing beneath insert

\[ F_{bst} = \zeta 0.567 f_{ck} S_q b_p L_3 \quad \text{Vertical force above billet} = 3538 \times 59.8 \times 10^{-3} = 211.4 \text{ kN and} \]

\[ A_{bst} = 81 \text{ mm}^2 \]

Use 1 no. H10 (78) say OK above insert

RHS design

\[ M_{\text{max}} = 26.15 \times 10^6 \text{ Nmm} \]

(ignores \( H = \mu V \) as this is resisted by surface friction over full contact area)
Beam-to-Column Connections: Column Insert

Work Example 5

Use $W_{pl} \geq M_{ez}/f_y$

and the web area (depth $d \times$ wall thickness $t$) is

$$2dt \geq V_{Ed}/0.6f_y$$

$$W_{pl} > 26.15 \times 10^3/275 = 95.1 \times 10^3 \text{ mm}^3$$

$$2dt > 200 \times 10^3/165 = 1212 \text{ mm}^2$$

Use $150 \times 100 \times 6.3$ RHS ($111 \text{ cm}^3$, $1890 \text{ mm}^2$)

Connecting dowel (Bending is ignored as the dowel is fully grouted in)

$$\mu = 0.4 \text{ for steel contact surface}$$

Horizontal force $H_{Ed} = 0.4 \times 200 = 80.0 \text{ kN}$ is carried by a dowel in double shear.

Maximum shear force (resolving horizontal reactions) = $(290/500) \times 80 = 46.4 \text{ kN}$

$$p_{bd} = 0.6p_{ybd} = 0.6 \times 512 = 307 \text{ N/mm}^2$$

Area $\geq 46.4 \times 10^3/307 = 151 \text{ mm}^2$

Bearing into $6.3 \text{ mm}$ thick RHS

Bolt diameter $\geq 46.4 \times 10^3/512 \times 6.3 = 14.4 \text{ mm}$

Use M16 grade 8.8 dowel (210) in 50 mm diameter tube ($= 16 + 2 \times 17 \text{ mm tolerances}$)
Beam-to-Column Connections: Column Insert

Work Example 6 (related to Work Example 5)

Determine the area of bursting reinforcement beneath the billet.

Assume:

\[ f_{yk} = 500 \text{ N/mm}^2. \]

\[ \zeta = \frac{F_{bst}}{V_{Ed}} = 0.25 \frac{(b - b_p)}{b} \]

The maximum compressive force occurs below the insert

\[ F = 0.567 f_{ck} S_d b_p (L_3 + L_2) \]

The horizontal bursting force is calculated from the end-block theory

\[ F_{bst} = \zeta F \]

\[ b_p = \text{insert breath} \]

\[ b_p / b = 100 / 300 = 0.33, \text{ then } \zeta = 0.167 \]

\[ F = 3638 \times (83.4 + 102.3) \times 10^{-3} = 675.6 \text{ kN} \]

\[ F_{bst} = 0.167 \times 675.6 = 112.8 \text{ kN} \]

\[ A_{bst} = 112.8 \times 10^3 / 0.87 \times 500 = 259 \text{ mm}^2 \]
Beam-to-Column Connections: Corbels

Deep corbels are usually around $H = 600-750$ mm, and so the inclination of the strut $\tan \beta$ is $> 2.5$.

Shallow corbels: Strut-and-tie analogy used in the design of corbels according to PD 6687, Annex B.4
Beam-to-Column Connections: Corbels

Further development in the design of corbels with high shear capacity or shear-with-negative moment capacity has used mechanical connections, such as steel shoes, or armour the concrete corbel heavily with steel.

(a) Beam end steel shoe on concrete corbel. (Courtesy of Peikko, Lahti, Finland.) (b) Cast in steel corbels showing transfer of (i) vertical force, (ii) horizontal force and (iii) torsion moment. (Courtesy of Peikko, Lahti, Finland.)
Beam-to-Column Connections: Corbels
Work Example 7

A 300 × 300 mm column supports a 10.0 m span 300 mm wide precast beam on a single-sided rc corbel. The beam has pretensioning strands extending to the end of the beam. The gap between the end of the beam and column is 15 mm. Because the beam experiences end rotations due to flexure of 0.005 rad, a dry neoprene bearing pad is to be used with its edge positioned 20 mm from the end of the beam. Given that the ultimate end reaction from the beam is 200 kN, determine the size of the bearing pad, size of the corbel and reinforcement required. Use $f_{ck} = 40 \text{ N/mm}^2$, $f_{yk} = 500 \text{ N/mm}^2$, $f_{neoprene} = 10 \text{ N/mm}^2$, $E_{neoprene} = 50 \text{ kN/mm}^2$. Cover to front and side face reinforcement = 30 and 25 mm at the top of the corbel. The service load (for crack control) may be $V_s = 150 \text{ kN}$. 

![Diagram of corbel connection]
Beam-to-Column Connections: Corbels
Work Example 7

Bearing pad

(Iteration is necessary due to beam end rotation influencing final pad stresses)

Without rotation effects,

Area $\geq 200 \times 10^3 / 10 = 20,000 \text{ mm}^2$

Maximum width $b_p = 300 - 2 \times \text{cover 30} = 240 \text{ mm}$, less tolerances $= 220 \text{ mm}$

$\therefore b_l \geq 20,000 / 220 = 91 \text{ mm}$.

Add about, say 30%, to this length to cater for end rotations.

Try 200 mm wide $\times$ 120 mm length $\times$ 10 mm bearing pad (considered to be a minimum thickness practically)

Then, $A = 24 \times 10^3 \text{ mm}^2$ and $Z = 200 \times 120^2 / 6 = 480 \times 10^3 \text{ mm}^3$

Additional eccentricity $e$ due to end rotation $= \theta$

$$e = \frac{0.005 \times 480 \times 10^3 \times 50 \times 120}{2 \times 200 \times 10^3 \times 10} = 3.6 \text{ mm}$$

Maximum stress in

$$\sigma_{Rd,2} = 0.85 \left(1 - \frac{f_{ck}}{250}\right) f_{ck} / 1.5$$

$\sigma_{Rd,2} = 0.85 \times (1 - 40 / 250) \times 40 / 1.5 = 19.04 \text{ N/mm}^2 > 10 \text{ N/mm}^2$

Use $200 \times 120 \times 10$ bearing pad
Beam-to-Column Connections: Corbels

Work Example 7

Distance to the centre of pad $a_c = \text{Gap} + \text{edge distance} + \text{half pad} + \Delta a_3$ (only as beam tendons are exposed) $= 15 + 20 = 120/2 + 10,000/2,500 = 99$ mm, say $a_c = 100$ mm
Assume 16 mm dia. top bars in a corbel
Corbel bearing ledge length from the face of the column:

Table for precast support $\Delta a_3 = \min\{10,10,000/1,200\} = 10$ mm

$a = a_c + \text{half pad} + \Delta a_3 + \text{bend allowance} r_o + \text{face cover}$
$= 100 + 60 + 10 + (4 \times 16) + 30 = 264$ mm.

Use $a = 265$ mm.

<table>
<thead>
<tr>
<th>Supporting material</th>
<th>$\sigma_{ed}/f_{cd}$</th>
<th>$a_3$ (mm)</th>
<th>$\Delta a_2$ (mm)</th>
</tr>
</thead>
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<tr>
<td></td>
<td>$&lt; 0.15$</td>
<td>$0.15-0.4$</td>
<td>$\geq 0.4$</td>
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<td>Steel</td>
<td>line</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>concentrated</td>
<td>5</td>
<td>10</td>
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<tr>
<td>Reinforced concrete</td>
<td>line</td>
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<td>10</td>
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<tr>
<td>$f_{cd} \geq 30$ N/mm$^2$</td>
<td>concentrated</td>
<td>10</td>
<td>15</td>
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<tr>
<td>Plain and reinforced concrete $f_{cd} &lt; 30$ N/mm$^2$</td>
<td>line</td>
<td>10</td>
<td>15</td>
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<tr>
<td>$f_{cd} \geq 30$ N/mm$^2$</td>
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<td>20</td>
<td>25</td>
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<tr>
<td>Precast concrete assume</td>
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<td>concentrated</td>
<td>10</td>
<td>15</td>
</tr>
</tbody>
</table>

Note: BS EN 1992-1-1, Tables 10.3 and 10.5.
Beam-to-Column Connections: Corbels

Work Example 7

To determine $h_c$ using $\theta = 30^\circ$
Depth of the front face

$$h_y \geq 25 + 64^* \text{ bend} + 50 \text{ straight} + (64 + 30)\tan(90^\circ - 30^\circ)/2 = 220 \text{ mm}$$

Try $h_c$ between the limits

$$h_c \leq h_y/0.5 = 440 \text{ mm}$$

or

$$h_c \geq 220 + 265\tan 30^\circ = 373 \text{ mm}$$

or $a_c/h \leq 0.5$, then $h \geq 100/0.5 = 200 \text{ mm}$

or

$$h_c \geq 1.2a = 1.2 \times 265 = 318 \text{ mm}$$

Therefore, use $h_c = 370 \text{ mm}$, making the soffit slope $\theta = \tan^{-1}150/265 = 29.5^\circ$

$$d = 370 - 25 \text{ cover} - 8 \text{ bar rad.} = 337 \text{ mm}$$
**Beam-to-Column Connections: Corbels**

**Work Example 7**

Then, \( a_i h_c = \frac{265}{370} = 0.71 \), just outside the \( f_i b \) suggested limit of 0.7, and \( a_i/d = \frac{100}{337} = 0.3 < \) suggested limit of 0.4. This will make the strut angle \( \beta \) quite steep.

\[ z_o > a_e, \quad \text{or} \quad d > a_e/0.75 = \frac{100}{0.75} = 133 \text{ mm} < 337 \text{ mm} \text{ OK} \]

**Shear check**

\[ v_{Rd} = 0.5 \times 0.6 (1 - 40/250) \times 40/1.5 = 6.72 \text{ N/mm}^2 \]

\[ d \geq 200 \times 10^3/300 \times 6.72 = 99 \text{ mm} < 337 \text{ mm} \text{ OK} \]

**Reinforcement design**

Then, if \( X = 0.5d \) in the limit, \( \beta = \tan^{-1}(0.75 \times 337/100) = 68.4^\circ (\tan \beta = 2.52 \text{ just} > 2.5) \)

\[ F_e = \frac{200}{\sin 68.4^\circ} = 215.1 \text{ kN} \]

\[ \sigma_{Rd,max} = 0.6 \times (1 - 40/250) \times 40/1.5 = 13.44 \text{ N/mm}^2 \]

\[ F_{cR} = 13.44 \times 300 \times 0.5 \times 337 \times \cos 68.4^\circ \times 10^{-3} = 250.1 > 215.1 \text{ kN} \]

Horizontal tie force \( F_{tt} = \frac{200}{\tan 68.4^\circ} = 79.2 \text{ kN} \)
**Beam-to-Column Connections: Corbels**

**Work Example 7**

\[ \mu = 0.7 \] for dry elastomeric bearing with vertical dowel(s) between the corbel and the beam

Horizontal frictional force \( F_{t2} = 0.7 \times 200 = 140.0 \text{ kN} \)

Total \( F_t = 219.2 \text{ kN} \)

\[ A_t = \frac{219.2 \times 10^3}{0.87 \times 500} = 504 \text{ mm}^2 \]

\[ A_{t,\text{min}} = 0.2\% \times 300 \times 370 = 222 \text{ mm}^2 \]

Use 3 no. H16 (603) top bars bent around the front face and anchored into the column

Spacing between bars = \( \frac{300 - 2 \times 54^\circ}{2} = 96 \text{ mm} < 225 \text{ mm} \)

for service \( V_s = 150 \text{ kN}, \sigma_s = (150/200) \times (504/603) \times 0.87 \times 500 = 218 \text{ N/mm}^2 \), for \( e_u = 0.3 \) then \( s \leq 225 \text{ mm} \)

Ratio of \( A_{t,\text{req}}/A_{t,\text{prov}} = 504/603 = 0.836 \)

\[ A_{t,\text{link}} \geq 0.5A_{t,\text{min}} = 0.5 \times 504 = 252 \text{ mm}^2 \]

to be forwarded to shear link calculation
Beam-to-Column Connections: Corbels

Work Example 7

Eccentricity $a_e = a_c + x/2$

$\tan \beta = z_o/a_e$

$h_y \geq 0.5h_c$

(Check angle $\beta$ according to next Figure as)

$$a_c = 100 + 200 \times 10^3 / (2 \times 13.44 \times 300) = 125 \text{ mm},$$

$$z_o = 337 - 79.2 \times 10^3 / (2 \times 13.44 \times 300) = 327 \text{ mm}.$$  \(\beta = \tan^{-1} 310/125 = 69^\circ\) compared to $68.4^\circ$ above.)

Anchorage length check

$$f_{cd} = 0.7 \times 0.3 \times 40^{2/3} / 1.5 = 1.64 \text{ N/mm}^2$$

$$f_{bd} = 2.25 \times 0.7 \times 1.0 \times 1.64 = 2.58 \text{ N/mm}^2$$

$$l_{bd,req} = 0.25 \times 16 \times 0.87 \times 500 \times 0.836 / 2.58 = 564 \text{ mm}$$

$$\alpha_1 = 0.7 \text{ (edge cover } C_d = 30 + \text{ say links } 8 + 16 = 54 > 3 \text{ dia.) and } \alpha_2 = 1.0$$

$$l_{bd} = 0.7 \times 1.0 \times 564 = 395 \text{ mm}$$

$$l_s = \text{ pad} = 120 \text{ mm}, \text{ then } l_{bd} - l_s = 395 - 120 = 275 \text{ mm}$$
Beam-to-Column Connections: Corbels

Work Example 7

\[ F_{bt} = 0.87 \times 500 \times 201 \times 0.836 \times \frac{(275/395) \times 10^{-3}}{2} = 50.9 \text{ kN} \]

\[ r_i = F_{bt} \left[ \frac{1}{a_b} + \frac{1}{2\Phi} \right] / 2f_{cd} \]

\[ a_b = 30 + \text{link bend} 2 \times 8 + 16/2 = 54 \text{ mm} \]

\[ r_i = 50.9 \times 10^3 \times \left[ \frac{1}{54} + \frac{1}{2 \times 16} \right] / (2 \times 40/1.5) = 47.5 \text{ mm} \text{ use} \]

\[ 3\Phi = 48 \text{ mm} \]
Beam-to-Column Connections: Corbels

Work Example 7

Use 3 no. H16 with 48 mm inner bend radius

Anchorage length from the face of pad = 275 mm \(<\) 300 + 40–30–8 = 302 mm, therefore provide 5Φ hook end or to at least tie off the three links.

**Shear reinforcement**

\[
a_c/2d = 40/2 \times 337 < 0.06 \text{, use } 0.25
\]

\[
A_{sw} = 0.25 \times 200/0.87 \times 500 = 115 \text{ mm}^2
\]

Minimum links calculated over a depth at the front face = 220 m

\[
A_{sw,\text{min}} = 0.08 \times 300 \times 220 \times \sqrt{40/500} = 67 \text{ mm}^2
\]

But, \(A_{s,\text{link}} \geq 252 \text{ mm}^2\)

\[
\text{Max } A_{sw} = \text{max}\{115, 67, 252\}
\]

Use 3 no. H8 (300 mm\(^2\)) at 35 mm centres.

Vertical links not required as \(a_c = 100 \text{ mm} < 0.5b_c = 185 \text{ m}\)

**Lateral bursting**

\[
b_p = 200 \text{ mm bearing pad, } b = 300 \text{ mm}
\]

\[
\zeta = 0.25 \times (300 - 200)/300 = 0.083
\]

\[
F_{\text{lat}} = 0.083 \times 215.1 = 17.9 \text{ kN}
\]

\[
A_{\text{lat}} = 17.9 \times 10^3/0.87 \times 500 = 41 \text{ mm}^2 < 150 \text{ mm}^2 \text{ (50\% link area) across the front face.}
\]
Beam-to-Column Connections:  
Beam End Design

1. **Shallow recess** is where a compressive strut resists the total beam end reaction

2. **Deep recess**, is where the depth of the nib above the bearing surface is insufficient to enable the strut to resist $V_{Ed}$, and so bent-up bars resist up to 50% $V_{Ed}$ and the compressive strut and vertical links the remainder.

In some instances, a prefabricated shear box partially or wholly replaces the stirrup cage.
Beam-to-Column Connections: Beam End Design: Shallow recess

Although codes do not give an explicit definition of a shallow recess, it is found that where $a_e \leq 0.6d_h$, truss action develops by providing adequate compression and tensile members inside the beam without the need for bent-up bars. Another rule of thumb is that the height of a shallow recess is $\frac{1}{3}$ to $\frac{1}{2}$ the beam depth, but it is $d_h$ that is more critical.
Beam-to-Column Connections: Beam End Design: Deep Recess

Where the depth of the recess is more than $2d/3$, or where $a_v > 0.6d_h$, or where $d_h$ is less than about 200 mm

Deep recess beam end design. (a) Force resolution; (b) rebar arrangement.
Beam-to-Column Connections:  
**Beam End Design: Steel Shear Box Concept**

A prefabricated shear box partially or wholly replaces the stirrup cage.

(a) strut and tie model  
(b) general arrangement of shear box with holding down rebars  
(c) general arrangement of shear box with holding down strap.
Beam-to-Column Connections:
Design Concepts for beam end shear

Structural action and typical reinforcement arrangements. (a) Truss action in half joint with deep pocket [4.16]; (b) truss action in half joint with shallow pocket [4.16]; (c) reinforcement positions.
Beam end shear design

Beam end shear reinforcement is required in a 10.0 m long x 600 mm deep by 300 mm wide beam to carry an ultimate shear force of 200 kN. The shape of the end of the beam is to be recessed across the full width of the beam to a depth of 200 mm and length 130 mm. A steel bearing plate is to be used in the horizontal bearing surface. The width of the supporting steel billet is 80 mm.

Given the following data, design the end shear reinforcement and bearing plate. The peripheral tie force is taken care of elsewhere. Use concrete grade C32/40, $f_{yk} \{f_y\} = 500 \text{ N/mm}^2$, $f_{yk} \{f_y\} = 250 \text{ N/mm}^2$ for welded bars, $p_y \{p_y\} \text{ for plate} = 275 \text{ N/mm}^2$ for Grade 43 steel, $f_{yd, w} = 275/1.25 = 220 \text{ N/mm}^2$, $f_{weld} = 215 \text{ N/mm}^2$ for 6 mm CFW. Cover to links = 25 mm. Allow an end tolerance for factory positioning the plate = 10 mm.

Table 4.11(a) Bursting force coefficients to BS EN 1992-1-1 (code uses $a/b$ instead of $b_p/b$).

<table>
<thead>
<tr>
<th>$b_p/b$</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi$</td>
<td>0.175</td>
<td>0.15</td>
<td>0.125</td>
<td>0.10</td>
<td>0.075</td>
</tr>
</tbody>
</table>

Table 4.11(b) Bursting force coefficients to BS 8110 (code uses $y_{po}/y_o$ instead of $b_p/b$).

<table>
<thead>
<tr>
<th>$b_p/b$</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi$</td>
<td>0.23</td>
<td>0.20</td>
<td>0.17</td>
<td>0.14</td>
<td>0.11</td>
</tr>
</tbody>
</table>
Figure 4.39  Details
(a) Strut and tie forces used and (b) reinforcement cage.
Beam-to-Column Connections:
Beam End Shear Design. Work Example 8

**Step 1: Bearing plate width**

\[ b/3 \leq b_p \leq b - 100 \text{ mm or } 0.4 \cdot b \]

Try \( b_p = 100 \text{ mm} \)

To determine \( S_q \) try \( b_l = 80 \text{ mm} \).

Resistance width = 100 + 2 \times 100 \text{ either side} = 300 \text{ mm}.

Resistance length = 80 + 80 \text{ at end of beam} = 160 \text{ mm}.

\[ S_q = \sqrt{\frac{300 \times 160}{100 \times 80}} = 2.45 < 3 \]

A steel plate design ultimate average bearing stress = \( 0.8 \cdot f_{cu} = 32 \text{ N/mm}^2 \)

\[ b_l > \frac{200 \times 10^3}{100 \times 44.4} = 45.0 \text{ mm} \]

Allow 10 mm end tolerance as given, plus (See clause 10.9.5.2) \( \Delta a_3 = 10000/2500 = 4 \text{ mm} \)

\( \Delta a_2 = 10 \text{ mm} \)

\[ b_l \geq 45 + 10 + 4 + 10 = 69 \text{ mm} \]

Try 80 mm as a minimum < 130 available

Effective plate bearing length, \( = 80 - 24 = 56 \text{ mm} \)

---

**Eurocode**

**BS8110**

A steel plate design ultimate average bearing stress = 0.8 \( f_{cu} = 32 \text{ N/mm}^2 \)

\[ b_l > \frac{200 \times 10^3}{100 \times 32} = 62.5 \text{ mm} \]

Allow 10 mm end tolerance as given, plus a further 10 mm fixing tolerance.

\[ b_l \geq 62.5 + 10 + 10 = 82.5 \text{ mm} \]

Try 90 mm < 130 available

Effective plate bearing length = 90 - 20 = 70 mm
Step 2: Plate thickness

The plate is subjected to an axial tension force $\mu V = 0.4 \times 200 = 80\, \text{kN}$

Cross-sectional area of plate $\frac{80 \times 10^3}{275} = 291\, \text{mm}^2$, so $t > \frac{291}{100} = 2.9\, \text{mm}$

The plate is also subjected to double shear of design stress $0.6\, p_{yk}$ \{0.6 $p_y$\} from the steel billet.

$\frac{200 \times 10^3}{2 \times 0.6 \times 275 \times 56} = 10.8\, \text{mm}$

Total $t > 2.9 + 10.8$

= 13.7 mm

> 12 mm preferred, therefore increase $b_t$ to 90 mm, and effective $b_t = 66$, such that $t = 2.9 + 9.2$

Use 100 x 90 x 12 plate

$\frac{200 \times 10^3}{2 \times 0.6 \times 275 \times 70} = 8.7\, \text{mm}$

Total $t > 2.9 + 8.7$

= 11.6 mm

Use 100 x 90 x 12 plate
Beam-to-Column Connections:
Beam End Shear Design. Work Example 8

Step 3: Reinforcement

Using the compressive strut-and-tensile-tie analogy, shown in Figures 4.39(a), it is first necessary to determine the angle $\theta$ of the primary strut $C_1$. From Figure 4.39(a), assuming 8 mm links, 12 mm top bars and 20 mm bars welded to the plate, $d = 400 - 25 - 8 - 12/2 - 12 - 20/2 = 339$ mm, and the reaction point at $1/2$ plate length $a_v = 130 + 25 + 8/2 - 90/2 = 114$ mm. $\theta = \tan^{-1}(114/339) = 18.6^\circ$.

Compressive strut force $C_1 = V/cos \theta = 200/0.948 = 211.0$ kN.

The width of the compressive strut $w_1$ must be checked to ensure that it does not extend beyond the end of the beam. The depth of the compressive block is also limited to $0.6d_h \{0.5d_h\}$, where $d_h$ is the effective depth to the horizontal tie bars attached to the plate. Referring to Figure 4.39(a), $d_h = 400 - 12 - 20/2 = 378$ mm.

Hence the maximum value of $w_1 = 0.6 \times 378 \times \sin \theta = 72.3$ mm

The actual value of $w_1$ is given by limiting the concrete stress to $0.6(1 - f_{ck}/250) f_{ck}/1.5 = 11.16$ N/mm$^2$

$$w_1 > \frac{211.0 \times 10^3}{300 \times 11.16} = 63$ mm $< 72.3$$

Hence the maximum value of $w_1 = 0.5 \times 378 \times \sin \theta = 60.3$ mm

The actual value of $w_1$ is given by limiting the concrete stress to $0.4f_{cu} = 16.0$ N/mm$^2$

$$w_1 > \frac{211.0 \times 10^3}{300 \times 16.0} = 44.0$ mm $< 60.3$$
Beam-to-Column Connections:
Beam End Shear Design. Work Example 8

The effect of the reaction creates two types of tensile force. The first is a lateral bursting force across the end of the beam and is a function of the width of the plate divided by the width of the beam, i.e. end block theory mentioned in the text.

Using $b_p/b = 0.33$ (see Table 4.11b), the bursting force is given by

$F_{ bursting} = 0.167 \times 200 = 33.3 \text{kN}$

and the area of the horizontal bars to resist this force is:

$A_{ bursting} = \frac{33.3 \times 10^3}{0.87 \times 500} = 77 \text{mm}^2$

Using $b_p/b = 0.33$ (see Table 4.11a), the bursting force is given by

$F_{ bursting} = 0.22 \times 200 = 44 \text{kN}$

and the area of the horizontal bars to resist this force is:

$A_{ bursting} = \frac{44 \times 10^3}{0.87 \times 500} = 101 \text{mm}^2$

Use two H8 bars at 100 mm centres, i.e. within the lower half of the end face of the beam. Also provide 77 mm$^2$ [101 mm$^2$] of vertical links above the plate; use two H12 links (diameter to be same as main links later).

The second force is a longitudinal tie force as given by $F_h = \mu V + V \tan \theta = (\mu + \tan \theta) V$.

Thus: $F_h = (0.4 + 0.336) \times 200 = 147.3 \text{kN}$

The horizontal bar is welded to the steel plate using a strength $f_y \left\{ f_{yk} \right\} = 250 \text{N/mm}^2$, even though a high-tensile bar is used.

$A_h = \frac{147.3 \times 10^3}{0.87 \times 250} = 667 \text{mm}^2$

Use two R25 bars welded to the plate using electrodes Grade E43.

The bars must have a bond length of 40 [35] diameters = 1000 mm [875 mm] beyond the intersection node with compressive strut $C_2$. Because of the remoteness of the mid-span tension reinforcement, 1.5 times this is often provided.
Beam-to-Column Connections:
Beam End Shear Design. Work Example 8

The length of a 6 mm double-sided fillet weld required to resist the rebar force distributed equally between two bars is:

\[ l_w = \frac{147.3 \times 10^3}{4 \times 6 \times 220} \]
\[ = 27.9 \text{ mm} \]
\[ l_w = \frac{147.3 \times 10^3}{4 \times 6 \times 215} \]
\[ = 28.5 \text{ mm} \]

plus 12 mm run-outs.

Use 50 mm weld length.

Figures 4.39
Beam-to-Column Connections:
Beam End Shear Design. Work Example 8

The vertical force \( V \) in the stirrups gives
\[
A_v = \frac{200 \times 10^3}{0.87 \times 500} = 460 \text{ mm}^2.
\]

Use three H12 links at 50 mm centres, the first of which should be placed as close to the end of the beam as possible, i.e. one cover distance. This means that two additional longitudinal bars (say two H12 bars) must be placed in the bottom corners of the beam to carry the first stirrup.

The horizontal compressive force \( H \) in the top of the beam is:
\[
H = V \tan \theta = 200 \times 0.336 = 67.3 \text{ kN}
\]

\[
A'_v = \frac{67.3 \times 10^3}{0.87 \times 500} = 155 \text{ mm}^2
\]

Use two H12 bars.

Anchorage length of bar required from nodal point = 40 [35] \( \times \) 12 = 480 mm [420 mm], or length of bar from end of beam = 420 + 155 = 635 mm [575 mm].

The maximum diagonal compressive force \( C_2 \) is given by \( V/\sin 45^\circ = 282.8 \text{ kN} \).

Using the same method as before to check confinement of the compressive strut, where \( d = 550 \text{ mm} \)

\[
w_{l,\text{max}} = 0.6 \times 550 \times \sin 45^\circ = 233 \text{ mm} \quad w_{l,\text{max}} = 0.5 \times 550 \times \sin 45^\circ = 194 \text{ mm}
\]

Actual width required \( w_i > \frac{282.8 \times 10^3}{300 \times 11.16} = 84.5 \text{ mm} \quad \text{Actual width required}. \quad = \frac{282.8 \times 10^3}{300 \times 16.0} = 58.9 \text{ mm}
\]

Figure 4.39(b) shows the reinforcement cage.

Finally the tension in the bottom of the beam is \( T = V \) (because the compressive strut \( C_2 \) is assumed to act at 45\(^\circ\)). Hence
\[
A_t = 460 \text{ mm}^2
\]

Use two H20 bars, with an internal radius of 60 [70] mm.

Anchorage length of bar from nodal point = 40 [35] \( \times \) 20 = 800 mm [700 mm]. These bars should be properly anchored, either to a corner angle using a fillet weld designed as before (for direct tension), or the bars should be fully anchored using a standard end hook for simply supported beams. Note that the area of this steel may be increased owing to the curtailment requirements of the main flexural reinforcement.
Column in Pocket Connection to Foundations

1. Pocket foundations. (a) Structural models in pocket foundations and (b) failure modes in pocket foundations.
Column in Pocket connection to foundations

Eurocode 2 Design
Column in Pocket connection to foundations

Eg. 7.126

Column pocket design

Design a column-to-pocket foundation connection required to support a 300 x 300 mm column subjected to an ultimate axial force of $N = 950\, \text{kN}$ ($N = 1000\, \text{kN}$) and a moment $M = 95\, \text{kNm}$ ($M = 100\, \text{kNm}$). The pocket has a 5° taper.

Use grade C40/50 concrete for the precast column, grade C30/37 for in situ infill, and grade C20/25 for the foundation, with $f_{yk} f_y = 500\, \text{N/mm}^2$ for the reinforcement. Cover to column reinforcement = 35 mm, and cover to foundation reinforcement = 50 mm.

Solution

Step 1: Column main steel and concrete design

$e = 100 \times 10^3/1000 = 100\, \text{mm} = h/3$, hence tension will develop in the column reinforcement.

Assume 16 mm dia. main bars and 10 mm dia. links. $D = 300 - 35 - 10 - 16/2 = 247\, \text{mm}$

Column design: $d_2/h = 0.176$, use BS EN 1992-1-1 derived column design chart $d_2/h = 0.2$.

$N/bh f_k = 0.264$ and $M/bh^2 f_k = 0.088$

$A_e = 0.08 \, bh f_k / f_{yk} = 576\, \text{mm}^2$

Use four H16 (804 mm²) with H8 links at 175 mm centres.

Actual stress in bars

$= (576/804) \times 0.87 \times 500$

$= 312\, \text{N/mm}^2$

N=950kN, M=95kNm

Column design: $d/h = 247/300 = 0.823$

Use BS 8110 derived column design, equivalent to Part 3, Chart 47 [7.75].

$N/bh = 11.11$, and $M/bh^2 = 3.704$.

$A_e = 0.728\% bh$

Use four H16 (804 mm²) with H8 links at 175 mm centres.

Actual stress in bars

$= (655/804) \times 0.87 \times 500$

$= 354\, \text{N/mm}^2$

N=1000kN, M=100kNm
Column in Pocket connection to foundations

Eg. 7.126

(cl.8.4, Anchorage bond length
for bars in tension, \( h > 250 \text{ mm} \).
Casting conditions for top bars
= poor
\( f_{bd} = 2.25 \times 0.7 \times 1 \times 0.14 \times 40^{3/4} \)
\( = 2.58 \text{ N/mm}^2 \)
(See cl. 8.4.4.) \( l_{bd,req} = 0.25 \times 312 \times 16/2.58 \)
\( = 484 \text{ mm} \)
(See cl. 8.4.4., Table 8.2) \( \alpha_1 = 1 \)
\( \alpha_2 = 1 - 0.15 \times (45 - 16)/16 = 0.728 \)
\( l_{bd} = 0.728 \times 484 = 352 \text{ mm} \)
Hence \( l = 50 + 352 + 35 \text{ cover} = 437, \)
say 450 mm.
\( \mu = 0.3, h = 0.3 \text{ m}, e = 0.1 \text{ m} \)
(7.90 - c) \( F_s = 950/1.09 = 872.0 \text{ kN} \)
Resultant = \( \sqrt{872.0^2 + 261.6^2} = 910.0 \text{ kN} \)
\( f'_e \geq 910.0 \times 10^3/(300^2 \times 0.482) \)
\( = 21.0 < 30 \text{ N/mm}^2 \) provided.
(7.90-d) \( F_s = (950 \times 0.25 - 872 \times 0.1635)/0.45 \)
\( = 211.0 \text{ kN} \)
\( f'_e \geq 211 \times 10^3/(0.2 \times 450 \times 300 \times 0.482) \)
\( = 16.2 < 30 \text{ N/mm}^2 \) provided.

Anchorage bond length for bars in
tension = \( 0.25 \times 354 \times 16/3.535 \)
\( = 401 \text{ mm} \)
Hence \( l = 50 + 401 + 35 \text{ cover} \)
\( = 486, \text{ say } 500 \text{ mm} \)
Then: \( L_d = \max(0.9 \times 500; \)
\( 500 - 50) = 450 \text{ mm} \)
\( p = 0.4 \times 37 \times 300 = 4400 \text{ N/mm} \)
\( \mu = 0.35, h = 0.3 \text{ m} \)
(7.90-c) \( L_d - 0.555L_s + 0.0227 = 0 \)
\( L_s = 45 \text{ mm} \)
\( F_s = 4400 \times 0.045 = 198.0 \text{ kN} \)

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BS8110
Column in Pocket connection to foundations

Step 2: Column confinement steel

Using lowest value $F_{bst} = 0.075 \times 950 = 71.3$ kN

$A_{bst} = \frac{71.3 \times 10^3}{0.87 \times 500} = 164 \text{ mm}^2$ per 2 legs

Use two H8 (200 mm$^2$) links at 50 mm centres.

Using lowest value $F_{bst} = 0.11 \times 1000 = 110$ kN

$A_{bst} = \frac{110 \times 10^3}{0.87 \times 500} = 253 \text{ mm}^2$

Use three H8 (300 mm$^2$) links at 50 mm centres.

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Step 3: Reinforcement around foundation pocket

Bars to be placed in upper half of pocket, i.e. to a depth of 225 mm.

Horizontal force induced by taper

$= 950 \tan 5^\circ = 83.1$ kN

Horizontal steel

$A_{sv} = \frac{(83.1 + 210.0) \times 10^3}{0.87 \times 500} = 674 \text{ mm}^2$

Bars to be placed in upper half of pocket, i.e. to a depth of 250 mm.

Horizontal force induced by taper

$= 1000 \tan 5^\circ = 87.5$ kN

Horizontal steel

$A_{sv} = \frac{(87.5 + 198.0) \times 10^3}{0.87 \times 500} = 656 \text{ mm}^2$

Use three H12 (678 mm$^2$) links at 75 mm centres around pocket.

Also provide nominal vertical hanger bars to support confinement links, three H10 bars.

See Figure 7.126 for final details.

BS8110
Column in Pocket connection to foundations

Eg. 7.126

Figure 7.126  Details
Column Base Plate- Connections to Foundation

**Column base plate design**

Design a column base plate connection required to support a 300 × 300 mm column subjected to an ultimate axial force of $N_{Ed} = 950 \text{kN}$ \{N = 1000 kN\} and a moment $M_{Ed} = 95 \text{kNm}$ \{M = 100 kNm\}.

Use grade C30/37 for the grout, grade S275 \{43\} steel for the base plate.

**Solution**

Try a 500 × 500 mm plate with $L = 100\text{mm}$ overhang and $d' = L/2 = 50\text{mm}$

\[
e = 95 \times 10^3/950 = 100\text{mm}
\]

Then $k = \frac{95 \times 10^3 \times (100 + 500/2 - 50)}{500 \times 500^2 \times 30} = 0.076$

\[
X^2 - 2X \left(1 - \frac{50}{500}\right) + 3 \times 0.076 \times \frac{0.076}{0.85^2} = 0
\]

Hence $X = 0.197$

Because $X < N/0.482 f_{c} b d = 0.263$ then $F$ is negative, so:

\[
X = 1 - \frac{2e}{d} = 1 - \frac{2 \times 100}{50} = 0.6
\]

and

\[
f_{c} = \frac{950 \times 10^3}{500 \times 0.6 \times 500} = 6.33 \text{N/mm}^2
\]

Hence

\[
t = \sqrt{\frac{2 \times 6.33 \times 100^2}{275}} = 21.5 \text{mm}
\]

Use 500 × 500 × 25 mm mild steel base plate.  

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\[
e = 100 \times 10^3/1000 = 100\text{mm}
\]

Then $k = \frac{1000 \times 10^3 \times (100 + 500/2 - 50)}{500 \times 500^2 \times 37} = 0.065$

\[
X^2 - 2X \left(1 - \frac{50}{500}\right) + 5 \times 0.065 = 0
\]

Hence $X = 0.203$

Because $X < N/0.4 f_{c} b d = 0.27$ then $F$ is negative, so:

\[
X = 1 - \frac{2e}{d} = 1 - \frac{2 \times 100}{50} = 0.6
\]

and

\[
f_{c} = \frac{1000 \times 10^3}{500 \times 0.6 \times 500} = 6.67 \text{N/mm}^2
\]

Hence

\[
t = \sqrt{\frac{2 \times 6.67 \times 100^2}{275}} = 22.0 \text{mm}
\]
Credit Acknowledgements of References

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*Obayashi Corporation, Japan. 2018. LRV Precast Installation Method*

*PCI. 2010. Prestressed Concrete Institute, Design Handbook, 7th ed., PCI, Chicago, IL.*

Credit Acknowledgements of References


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